BIBLIOGRAPHY

1. Pontriagin, L. S., Boltianskii, V. G., Gamkrelidze, R. V. and Mishchenko, E. F., Mathematical Theory of Optimal Processes. M., Fizmatgiz, 1961.
2. Troitskii, V. A., Variational problems in the optimization of control processes in systems with bounded coordinates. PMM Vol. 26, №3, 1962.
3. Bryson, A. E. and Denhem, W. F., Optimal programing problems with inequality constraints, I. Necessary conditions for extremal solutions. AIAA Journal Vol. 1, №11, 1963.
4. McIntyre, J. and Paiewonsky, B., On optimal control with bounded state variables. Advances in Control Systems. Theory and Application. New York – London, Acad. Press, Vol. 5, 1967.

Translated by L. K.

SOME PROPERTIES OF PLANE AND THREE-DIMENSIONAL BODIES

PMM Vol. 34, №1, 1970, pp. 132-134

V. P. SDOBYREV
(Moscow)

(Received June 24, 1969)

We determine the relationships between the structural elements of geometric bodies represented as plane or three-dimensional systems in the form of hinged-rod or rigid hingeless lattices by considering the intersections and gaps of the latter in a plane field. Three-dimensional bodies can be investigated by projecting them on a plane. The projections considered in the present paper exclude complete coincidence of individual elements. One way of establishing the relationships between the elements of plane and three-dimensional bodies is by mathematical induction.

1. Initial assumptions. We begin by defining some terms:

Rods are straight or curvilinear bodies one of whose dimensions is large compared to the other. These bodies possess three degrees of freedom in a plane and five degrees of freedom in space.

Disks are bodies or geometrically nonvarying links with three degrees of freedom in a plane and six degrees of freedom in space.

Hingeless (free) intersections are domains or points of contact between elements (disks, rods, or both).

Hinged intersections are domains or points of contact between elements into which hinges have been introduced.

Gaps are individual closed domains within the outer contour whose dimensions can be determined by direct calculation of the ares of clearance.

We assume that the gaps can be of any shape, e. g. a uniangle (a domain bounded by a closed curve with a single acute or obtuse corner), a biangle (a domain bounded by a closed curve. with two acute or obtuse corners or one of each), a triangle, a polygon, or a nonangle (a domain bounded by a closed curve with smooth transitions from one curve to another).

2. Plane and three-dimensional bodies. The relationship between the geometric structural elements of models of plane and three-dimensional free bodies can be expressed in general form as

$$M - n_p^{\times} - \gamma = {}^1/_3\ (n_p^{\circ} + S_0 + \rho - 2n_0 - 3\ d) \tag{2.1}$$

$$n_p^{\times} = 1n_2^{\times} + 2n_3^{\times} + \ldots + (k - 1)\ n_k^{\times} \tag{2.2}$$

Here M is the number of gaps, $n_p^{\times}$ is the reduced number of hingeless (free) intersections, $n_k^{\times}$ is the number of k-tuple free intersections, k is the number of element ends forming a free intersection, and γ is the number of free bodies, systems, or elements. (By "free elements" we mean unattached, fixed-base combinations of disks, hinges (points), and rods (straight or curvilinear lines) formed by hingeless or hinged intersections, or both).

The reduced number n_p° of nodal or hinged intersections is given by

$$n_p^{\circ} = (0-1)\ n_0^{\circ} + (1 - 1)\ n_1^{\circ} + 1n_2^{\circ} + \ldots + (k - 1)\ n_k^{\circ} \tag{2.3}$$

where n_k° is the number of k-tuple nodal or hinged intersections, k is the number of element ends forming a nodal or hinged intersection, S_0 is the number of rods in the model of a geometric body, the number of lines constituting a geometric figure, or the number of ribs of the polyhedra forming the framework of a model, and $\rho = \rho_n + \rho' = 2\rho_0' + 3\rho_0$; ρ is the total number of internal connections, ρ_n is the number of internal connections over the hinged sections, ρ' is the number of internal connections over the disk sections, ρ_0 is the number of cuts over the disk sections required for the complete elimination of individual closed contours (gaps), ρ_0' is the number of necessary cuts over the hinged-disk sections, n_0 is the number of simple hinges (this applies to the nodes of a rod lattice exclusively), geometric points, nodes, or vertices of polyhedra; d is the number of disks.

We can transform relation (2.1) as follows:

$$\gamma^{-1}\ [M - n_p^{\times} - {}^1/_3(n_p^0 + S_0 + \rho - 2n_0 - 3d)] = \text{const} = 1 \tag{2.4}$$

The notation

$$M_{p\gamma} = M - n_p^{\times},\ M_p = {}^1/_3\ (n_p^{\circ} + S_0 + \rho - 2n_0 - 3d)$$

where $M_{p\gamma}$ is the reduced number of plane graphs and M_p is the reduced number of gaps, enables us to rewrite relation (2.1) as

$$\gamma^{-1}\,(M_{p\gamma} - M_p) = \text{const} = 1 \tag{2.5}$$

Let us formulate this relation as a theorem.

Theorem. The ratio of the difference between the reduced number of plane graphs $M_{p\gamma}$ and the reduced number of gaps M_p to the number of bodies γ for models of plane and three-dimensional geometric bodies is constant and equal to unity.

We provide relations (2.1)–(2.5) by mathematical induction. (We omit the proof from the present paper).

Corollary. It is easy to see that in the case of hinged-rod models, when [1]

$$\rho = d = 0,\quad 2M_p = n_p^{\circ} - n_0,\qquad 2S_0 = n_p^{\circ} + n_0$$

relation (2.1) can be rewritten as

$$M_p' = M - n_p^{\times} - \gamma = S_0 - n_0 = n_p^{\circ} - S_0 = {}^1/_2\ (n_p^{\circ} - n_0) \tag{2.6}$$

For models of geometric systems represented as rigid-lattice systems (the framework of a hingeless system for which $S_0 = n_0 = n_p^{\circ} = 0$) relation (2.1) is of the form

$$M_p'' = M - n_p^{\times} - \gamma = \rho_0 - d \quad (2.7)$$

It is clear that the reduced numbers of gaps for the same model (or group) in the two cases is

$$M_p = M_p' = M_p'' \quad (2.8)$$

If we limit ourselves to convex polyhedra for $\gamma = 1$, then Eq. (2.8) and the Decartes-Euler formula [2, 3] for the number of faces

$$\Gamma = S_0 - n_0 + 2 \quad (2.9)$$

imply that the difference $M - n_p^{\times}$ is the reduced number of plane graphs or the number (minus one) of faces of a complex polyhedron,

$$M - n_p^{\times} = \Gamma - 1, \qquad \Gamma = M - n_p^{\times} + 1 \quad (2.10)$$

This implies that

$$\Gamma = M - n_p^{\times} + \gamma = M_p + 2\gamma \quad (2.11)$$

when $\gamma > 1$.

Comparison of formulas (2.8) and (2.11) also yields the following relations for the number of faces of convex polyhedra:

$$\begin{aligned}\Gamma = M_p + 2\gamma = S_0 - n_0 + 2\gamma = n_p^{\circ} - S_0 + 2\gamma = \\ = {}^1/_2\,(n_p^0 - n_0) + 2\gamma = \rho_0 - d + 2\gamma\end{aligned} \quad (2.12)$$

(The first three relations of (2.12) were proved in 1953 [1]).

It is easy to show that in the case of mutual self-projection of models of convex polyhedra the formula for the number of faces can be written as

$$\Gamma = M_p + 2\gamma' \qquad M_p = M - n_p^{\times} - \gamma, \quad \gamma' \geqslant \gamma \quad (2.13)$$

Here γ is the number of free elements (the number of individual combinations of various geometric elements) and γ' is the number of individual polyhedra.

If $\gamma' = \gamma$, then formula (2.13) becomes formula (2.12). Let us consider some examples.

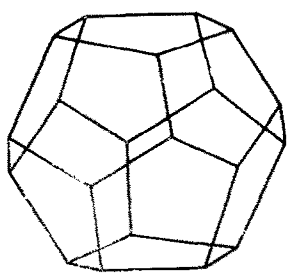

Fig. 1

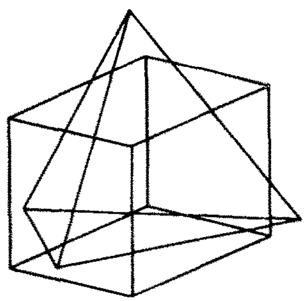

Fig. 2

Example 1. For the model of a dodecahedron shown in Fig. 1 we have

$$M = 19, \; n_p^{\times} = 8, \; \gamma = 1, \; n_p^{\circ} = 40, \; S_0 = 30, \; n_0 = 20$$
$$\min \rho_0 = 19, \; \min d = 9$$

According to relations (2.11) and (2.12) the number of faces is

$$\Gamma = 19 - 8 + 1 = 30 - 20 + 2 = 40 - 30 + 2 = {}^1/_2\,(40 - 20) + 2 = 19 - 9 + 2 = 12$$

Example 2. Let us verify relation (2.13) for a case of mutual self-projection of two polyhedra (a tetrahedron and a hexahedron; see Fig. 2). Here

$$M = 25, \qquad n_2^{\times} = 16, \qquad n_3^{\times} = 1$$

and according to relation (2.2)

$$n_p^{\times} = 1 \cdot n_2^{\times} + 2n_3^{\times} = 1 \cdot 16 + 2 \cdot 1 = 18$$

$$\gamma = 1,\ \gamma' = 2,\ n_p^{\circ} = 24,\ S_0 = 18,\ n_0 = 12,\ \min \rho_0 = 25,\ \min d = 19$$

The reduced number of gaps M_p given by relation (2.8) is

$$M_p = 25-18-1 = 18-12 = 24-18 = {}^1/_2\,(24-12) = 25-19 = 6$$

According to relation (2.13) the number of faces is

$$\Gamma = M_p + 2\gamma' = 6 + 4 = 10$$

BIBLIOGRAPHY

1. Sdobyrev, V. P., A Criterion of Nonvariability of Plane and Three-Dimensional Systems. Inzh. Sb. Vol. 15, (pp. 187-190), 1953.
2. Courant, R. and Robins, G., What is Mathematics? Moscow, "Prosveshchenie", (Russian translation), 1967.
3. Euler, L., Collected Papers in Honor of the 250-th Anniversary of His Birth Prepared by the Academy of Sciences SSSR, Izd. Akad. Nauk SSSR, (pp. 150-158) Moscow.

Translated by A. Y.

THE INSTANT OF FORMATION OF A SHOCK WAVE IN A TWO-WAY TRAFFIC FLOW

PMM Vol. 34, №1, 1970, pp. 135-137
G. P. SOLDATOV
(Saratov)
(Received December 6, 1967)

Two continuity equations describing a symmetric two-way traffic flow are considered. These equations are then used to find the instant of formation of a shock wave by the Riemann method.

The theoretical analysis of traffic flows has lately received much attention; theories of traffic flows have been constructed on the basis of analysis of motion of descrete objects (often called "motorcars"), mathematical statistics [1, 2], classical mechanics [3--5], and statistical mechanics [6]. Survey [5] contains a discussion of studies applying the hydrodynamic analogy to traffic flows. The first of the two current trends of research is based on the kinematic wave theory; the second is based on the Greenberg relation for continuous traffic flows. The kinematic waves traveling opposite to the direction of traffic and the distinctions between them and dynamic waves are considered in [3].

We make use of the hydrodynamic model of a two-way traffic flow. The traffic flow in this model is described by two continuity equations and by two empirical relationships between the velocities and densities of the flows of cars moving in opposite directions. The hydrodynamic model of a traffic flow enables us to predict the formation of shock waves and to analyze many cases of shock wave propagation in the flow. The basic properties of traffic flow are established in [4].